1. Mixed numbers and improper fractions



Name _____ Class _____ Date _____

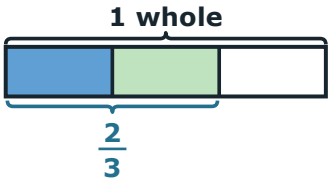
Max and Teddy's stop · Santorini · the blue domes. Max and Teddy have flown from Porto to Santorini, and Max reads his guidebook on the white steps above the blue domes while Teddy rolls onto his back for a tummy rub.

> **Today:** More parts than one whole make an **improper fraction**; the same amount is a **mixed number**, a whole number and a fraction: $\frac{7}{4} = 1\frac{3}{4}$.

Warm-up Wholes and parts

Each answer helps with the next one. Fill in the model, then write the equation.

1 **a)** Break $\frac{2}{3}$ into thirds.



$\frac{2}{3} = \frac{1}{3} + \frac{\square}{\square}$

b) Write $\frac{4}{3}$ in wholes and thirds.



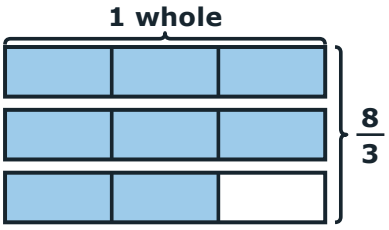
$\frac{4}{3} = \frac{3}{3} + \frac{\square}{\square} = \square\frac{\square}{\square}$

c) Write $\frac{6}{3}$ in wholes and thirds.



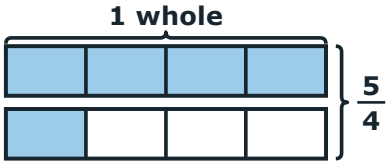
$\frac{6}{3} = \frac{3}{3} + \frac{\square}{\square} = _____$

d) Write $\frac{8}{3}$ in wholes and thirds.



$\frac{8}{3} = \frac{6}{3} + \frac{\square}{\square} = \square\frac{\square}{\square}$

e) Write $\frac{5}{4}$ in wholes and quarters.



$\frac{5}{4} = \frac{4}{4} + \frac{\square}{\square} = \square\frac{\square}{\square}$

f) Write $\frac{7}{4}$ in wholes and quarters.



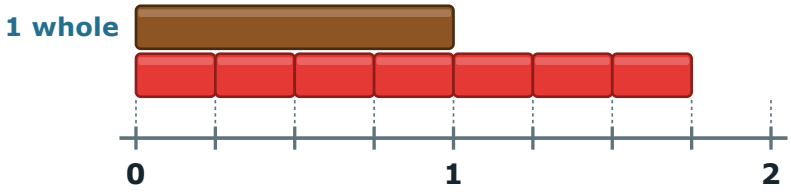
$\frac{7}{4} = \frac{4}{4} + \frac{\square}{\square} = \square\frac{\square}{\square}$

Mixed numbers and improper fractions

KNOWLEDGE Count the parts of each whole

WORKED EXAMPLE

The brown rod is 1 whole, and each red rod is $\frac{1}{4}$. Where does a train of 7 red rods end?



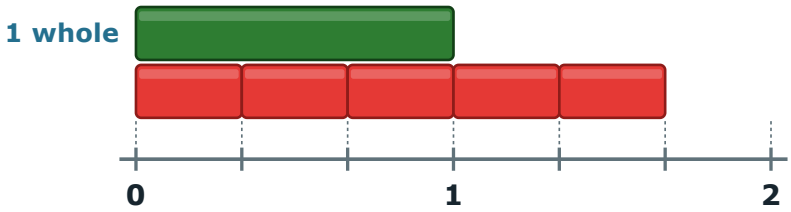
quarters in 1 whole: 4

past 1: $\frac{3}{4}$ 7 quarters = $1\frac{3}{4}$

1. 4 red rods make the brown rod: **4** quarters make 1 whole.
2. 7 red rods: 4 make 1 whole, and $\frac{3}{4}$ is past 1.
3. The train ends at $\mathbf{1\frac{3}{4}}$: $\frac{7}{4} = 1\frac{3}{4}$.

$$\frac{7}{4} = \frac{4}{4} + \frac{3}{4} = 1\frac{3}{4}$$

2 ★ The dark green rod is 1 whole, and each red rod is $\frac{1}{3}$. A train of 5 red rods is $\frac{5}{3}$. **How much is past 1? Write $\frac{5}{3}$ as a mixed number.**



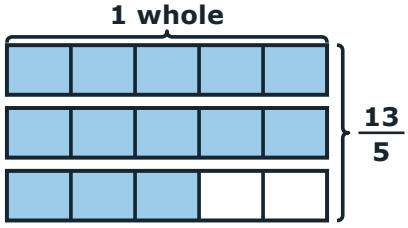
$\frac{5}{3} = 1 + \frac{\square}{\square} = \square\frac{\square}{\square}$

3 ★ Each Numicon 4-shape is 1 whole, and each peg is $\frac{1}{4}$. The pegs show $2\frac{1}{4}$. **How many quarters are in the 2 full shapes? How many quarters are there altogether?**



$2\frac{1}{4} = \frac{\square}{\square} + \frac{1}{4} = \frac{\square}{\square}$

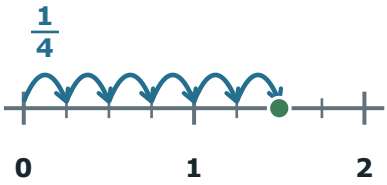
4 ★ Each bar is 1 whole, cut into fifths, and $\frac{13}{5}$ is shaded. **How many bars are full? Write $\frac{13}{5}$ as a mixed number.**



$\frac{13}{5} = _____ + \frac{3}{5} = \frac{\square}{\square}$

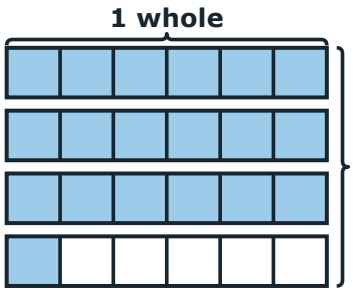
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5 ★ Count on in quarters from 0: **6** jumps of $\frac{1}{4}$. **Where do you land? Write it as a mixed number in quarters.**



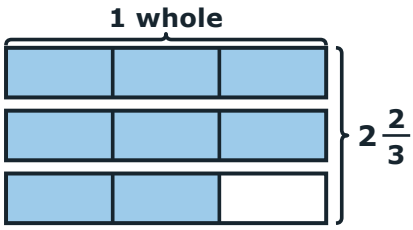
6 quarters = $\frac{\square}{\square}$ = $\square\ \frac{\square}{\square}$

6 ★ Each bar is 1 whole. **How much is shaded? Write it as an improper fraction and as a mixed number.**



shaded = $\frac{\square}{\square}$ = $\square\ \frac{\square}{\square}$

7 ★ The bars show $2\frac{2}{3}$. **How many thirds are in the 2 full bars? How many thirds is $2\frac{2}{3}$?**



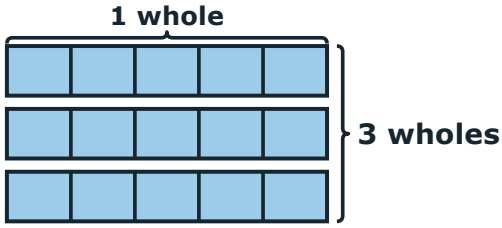
$2\frac{2}{3}$ = $\frac{\square}{\square}$ + $\frac{2}{3}$ = $\frac{\square}{\square}$

8 ★ **What does the arrow point to?** Write it as an improper fraction and as a mixed number.



the arrow = $\frac{\square}{\square}$ = $\square\ \frac{\square}{\square}$

9 ★ Each bar is 1 whole, cut into fifths. **How many fifths make 3 wholes?**



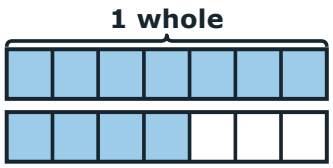
3 = $\frac{\square}{\square}$

10 ★ **What does the arrow point to?** Write it as an improper fraction and as a mixed number.



the arrow = $\frac{\square}{\square}$ = $\square\ \frac{\square}{\square}$

11 ★ Each bar is 1 whole, and the bars show $1\frac{4}{7} = \frac{11}{7}$. **Shade 1 more seventh, then 1 more. Write $1\frac{5}{7}$ and $1\frac{6}{7}$ as improper fractions.**



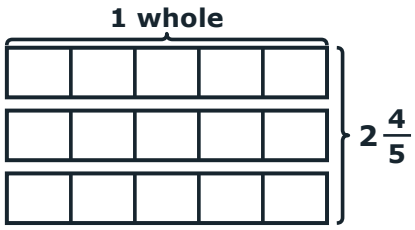
$1\frac{5}{7}$ = $\frac{\square}{\square}$

$1\frac{6}{7}$ = $\frac{\square}{\square}$

Mixed numbers and improper fractions

12 ★ Finish the model yourself, then write the equation.

a) Show $2\frac{4}{5}$ on bar models. **How many fifths is $2\frac{4}{5}$?**



1 whole = _____ fifths

$2\frac{4}{5} = \frac{\square}{\square}$

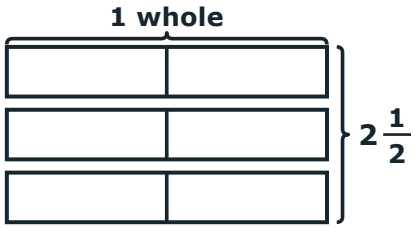
b) Show $\frac{23}{10}$ on bar models. **Write $\frac{23}{10}$ as a mixed number.**



1 whole = _____ tenths

$\frac{23}{10} = \square\frac{\square}{\square}$

c) Show $2\frac{1}{2}$ on bar models. **How many halves is $2\frac{1}{2}$?**



1 whole = _____ halves

$2\frac{1}{2} = \frac{\square}{\square}$

13 ★ Mark $\frac{13}{8}$ on a number line from 1 to 2. **Write $\frac{13}{8}$ as a mixed number.**



1 whole = _____ eighths

$\frac{13}{8} = \square\frac{\square}{\square}$

APPLICATION **On the steps of Santorini** Draw your own model, then write your number sentence.

14 ★★ Thalia fills **9** small jugs for the tables on the terrace, and Teddy gets a bowl of his own. There is $\frac{1}{2}$ ℓ of water in each jug. **How many halves make 1 litre? How many litres of water are in the 9 jugs? Write it as a mixed number.**



1 litre = _____ halves

_____ = $\square\frac{\square}{\square}$ ℓ

15 ★★ From Fira, Max and Teddy walk the cliff path above the blue domes. A marker stone stands at every $\frac{1}{3}$ km, and Teddy sniffs at each one. They stop at the **7th** marker stone. **How many thirds make 1 km? How far have they walked? Write it as a mixed number.**



1 km = _____ thirds

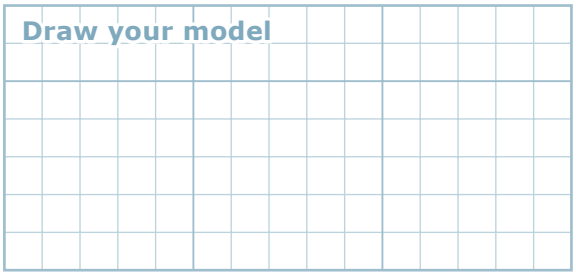
_____ = $\square\frac{\square}{\square}$ km

Mixed numbers and improper fractions

16 ★★

Thalia cuts each of her orange cakes into **6** equal slices. A school group on a trip to the blue domes eats **17** slices, and Teddy waits under the table for the crumbs. **How many slices make 1 cake? How many cakes does the group eat? Write it as a mixed number.**

Draw your model



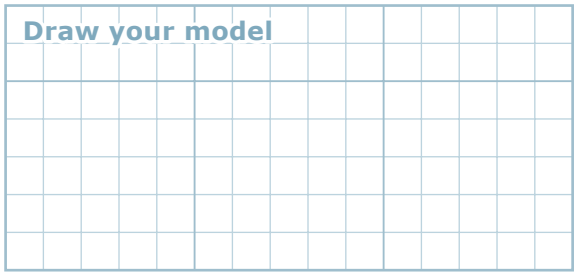
1 cake = _____ slices

17 slices = ☐ ☐/☐ cakes

17 ★★

Thalia bakes her spinach pies in trays and cuts each tray into **8** equal pieces. Teddy sniffs at the warm trays. At lunch time, **3** full trays and $\frac{5}{8}$ of a tray are left. **How many pieces make 1 tray? How many eighths of a tray are left? Write it as an improper fraction.**

Draw your model



1 tray = _____ pieces

_____ = ☐/☐ trays

REASONING More than one whole

Decide, then explain how you know.

18 ★★★

a) Max writes $\frac{13}{4} = 1\frac{3}{4}$ in his journal. He says: "The 1 of 13 is the whole number." **What mistake has Max made?**

- ◯ There is no mistake
- ◯ 4 quarters make 1 whole, so 13 quarters are 3 wholes and 1 quarter
- ◯ 13 quarters are 13 wholes

b) Now mark $\frac{11}{4}$ on a number line from 2 to 3. **How many quarters make 1 whole? Write $\frac{11}{4}$ as a mixed number.**



1 whole = _____ quarters

$\frac{11}{4}$ = ☐ ☐/☐

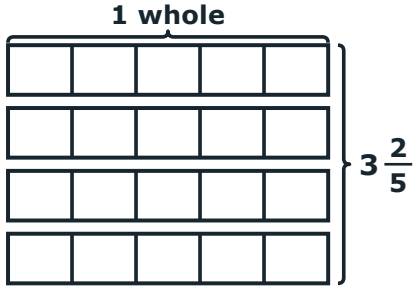
a) Explain: _____

19 ★★★

a) Max writes $3\frac{1}{5} = \frac{15}{5}$. Thalia says: "It is $\frac{16}{5}$." **Who is right?**

- ◯ Max: 3 wholes are 15 fifths
- ◯ Thalia: 3 wholes are 15 fifths, and 1 fifth more makes 16 fifths
- ◯ Both of them

b) Now show $3\frac{2}{5}$ on bar models. **How many fifths make 1 whole? How many fifths is $3\frac{2}{5}$?**



1 whole = _____ fifths

$3\frac{2}{5}$ = ☐/☐

a) Explain: _____

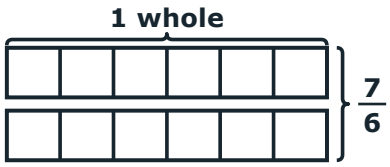
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20 ★★★

a) Thalia says: "When the numerator is greater than the denominator, the fraction is more than 1 whole." **True or false?**

◯ True ◯ False

b) Show $\frac{7}{6}$ on bar models. **Write $\frac{7}{6}$ as a mixed number.**



1 whole = _____ sixths

$\frac{7}{6}$ = ☐ $\frac{\square}{\square}$

a) Explain: _____

21 ★★★

a) Max writes $\frac{17}{3}$ in his journal. Thalia says: "$\frac{17}{3}$ is between 5 and 6." **Do you agree with Thalia?**

◯ Yes: 15 thirds make 5 and 18 thirds make 6, so 17 thirds are between them

◯ No: 17 is far more than 6

◯ No: it is between 1 and 2

b) Mark $\frac{17}{3}$ on a number line from 5 to 6. **How many thirds make 1 whole? Write $\frac{17}{3}$ as a mixed number.**



1 whole = _____ thirds

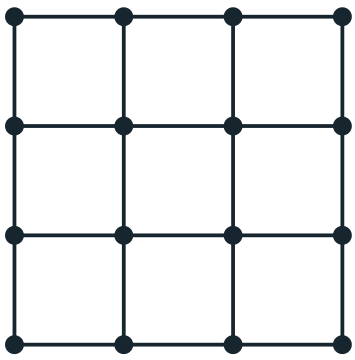
$\frac{17}{3}$ = ☐ $\frac{\square}{\square}$

a) Explain: _____

22 PROBLEM OF THE DAY **How many rectangles?**

On the steps above the blue domes of Santorini, Thalia brings Max a slab of chocolate, 3 blocks across and 3 down, with his lemonade. Teddy sniffs at it, but chocolate is not for dogs, so he gets a dog biscuit instead. Max marks out rectangles along the grooves of the slab: any size, anywhere on it. A square is a rectangle too.

How many rectangles are there on the slab altogether? Count every size in every place: two rectangles of the same size in different places are two rectangles.



Think further: How many are there on a slab 4 across and 4 down? Find a quick way.

Explain one thing you learned.

I can do this: ◯ independently ◯ with help ◯ I need more practice